Rigidity percolation, floppy modes, and frequency dependent shear modulus of colloidal gels

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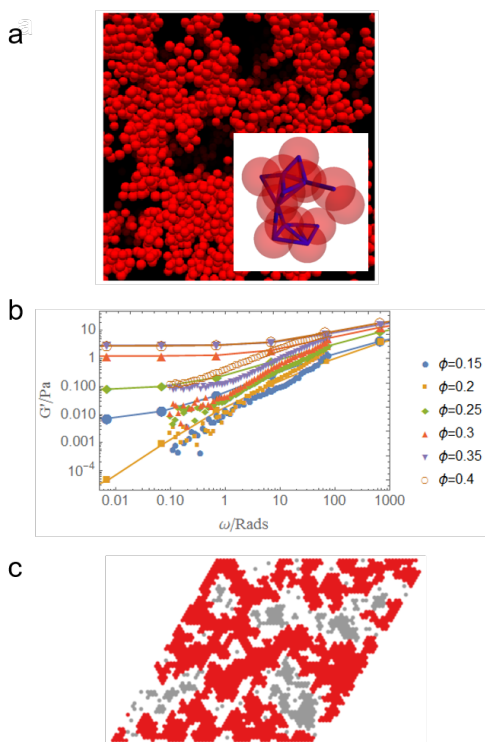[6] *Department of Physics, Georgetown University, Washington D.C. 20057*

FIG. 1. (a) Reconstructed 3D image of colloidal gel particles and bonds, from confocal image analysis. (b) Agreement between experimental (scattered data points) and simulation results (connected data points) of colloidal gel shear modulus as a function of frequency. (c) Numerical simulation of correlated site rigidity percolation (red: percolating rigid cluster, gray: floppy clusters).

Colloidal particles subject to short-range attractive depletion interactions form gels: self-organized, disordered networks with marginal solidity. In this talk, we mainly focus on mechanical stability of colloidal gels.

The notion of mechanical stability traces back to J. C. Maxwell, who relates the number of floppy modes in a system to the difference between the numbers of degrees of freedom and constraints. In Maxwell's theory, mechanical stability is equivalent to the vanishing of floppy modes, which simplifies to the equation $\langle z \rangle = 2d$ for simple systems with point-like particles and central-force constraints (where z is the particle coordination number and d is the spatial dimension).

We will first discuss our recent project taking real 3D configurations of colloidal particles from confocal microscopy (Fig. 1a), and numerically study their mechanical properties. We find that, interestingly, for the samples we studied with volume fraction within $(15\%, 40\%)$, the coordination number is significantly below $2d$. However, these samples still exhibit finite shear modulus. We performed a computational study of the mode structure of these samples, and obtained good agreement with experimental shear modulus at varying frequency [1].

We will then discuss our theoretical study on the formation of these dilute structures with mechanical stability. Our theoretical model is based on site rigidity percolation with positional correlation between the particles that mimics the short range attraction. We obtained a phase diagram for this simple model that resembles the phase diagram of colloidal gels. We have also analyzed the criticality of the rigidity transition in this model, and found that it belongs to the same universality class as uncorrelated rigidity percolation, albeit with shifted transition point, indicating lower critical volume fraction for stronger correlation [2].

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[1] D. Z. Rocklin, L. Hsiao, M. Szakasits, M. J. Solomon, X. Mao, *manuscript in preparation* (2017).

[2] S. Zhang, L. Zhang, D. Z. Rocklin, E. Del Gado, and X. Mao *manuscript in preparation* (2017).